NAMIBIA UNIVERSITY
OF SCIENCE AND TECHNOLOGY

FACULTY OF HEALTH, NATURAL RESOURCES AND APPLIED SCIENCES
SCHOOL OF NATURAL AND APPLIED SCIENCES
DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCE

QUALIFICATION: BACHELOR OF SCIENCE HONOURS IN APPLIED MATHEMATICS	
QUALIFICATION CODE: 08BSHM	**LEVEL: 8**
COURSE CODE: ADC801S	**COURSE NAME**: ADVANCED CALCULUS
SESSION: JUNE 2023	**PAPER:** THEORY
DURATION: 3 HOURS	**MARKS: 100**

FIRST OPPORTUNITY EXAMINATION QUESTION PAPER	
EXAMINER	**Prof A.S Eegunjobi**
MODERATOR	Prof O.D Makinde

INSTRUCTIONS
1. Answer ALL the questions. 2. Write clearly and neatly. 3. Number the answers clearly.

PERMISSIBLE MATERIALS

1. Non-programmable calculator without a cover

THIS QUESTION PAPER CONSISTS OF 3 PAGES (Including this front page)

1. (a) If $x = r\cos\theta$ and $y = r\sin\theta$, find the (r, θ) equations for ϕ which obeys Laplace's equation in two-dimensional caresian co-ordinates

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0$$

(5)

(b) if $Q = \log(\tan x + \tan y + \tan z)$, show that

$$\sin 2x \frac{\partial u}{\partial x} + \sin 2y \frac{\partial u}{\partial y} + \sin 2z \frac{\partial u}{\partial z} = 2$$

(5)

(c) If $u = x^2 \tan \frac{y}{x}$, find

$$\left.\frac{\partial^2 u}{\partial x \partial y}\right|_{(-1,2)}$$

(5)

2. (a) Minimize $f(x_1, x_2) = x_1 - x_2 + 2x_1^2 + 2x_1x_2 + x_2^2$ by taking the starting from the point $\mathbf{X_1} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$ using Davidon–Fletcher–Powell (DFP) method with

$$[B_1] = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \epsilon = 0.01$$

(10)

(b) Minimize $f(x_1, x_2) = x_1 - x_2 + 2x_1^2 + 2x_1x_2 + x_2^2$ by taking the starting from the point $\mathbf{X_1} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$, by using Newton's Method (10)

3. (a) If

$$\phi = x^n + y^n + z^n$$

show that

$$\mathbf{r} \cdot \nabla \phi = n\phi$$

where n is constant (8)

(b) Find the directional derivative of the function

$$\phi(x, y, z) = x^2 y - 3yz + 2xz$$

in the direction

$$\mathbf{n} = 4i - 7j + 4k$$

at the point $(3, -2, 1)$. (8)

4. (a) Determine the minimum distance between the origin and the hyperbola defined by $x^2 + 8xy + 7y^2 = 226$ (6)

(b) Show that $\nabla \cdot (\nabla g^m) = m(m+1)g^{m-2}$, if $\bar{g} = xi + yj + zk$. (9)

(c) A material body's geometric representation is a planar area R, delimited by the curves $y = x^2$ and $y = \sqrt{2 - x^2}$ within the boundaries $0 \leq x \leq 1$. The density function associated with this model is denoted as $\rho = xy$.

i. Find the mass of the body. (4)

ii. Find the coordinates of the center of mass. (5)

5. A curve is defined parametrically by

$$x(t) = ae^t \cos t, \quad y(t) = ae^t \sin t, \quad \text{and} \quad z(t) = \sqrt{2}a(e^t - 1).$$

Find the following for the curve:

(a) The tangent vector $\hat{\mathbf{T}}$ (5)

(b) The curvature κ (5)

(c) The principal normal vector $\hat{\mathbf{N}}$ (5)

(d) The binormal vector $\hat{\mathbf{B}}$ (5)

(e) The torsion τ (5)

End of Exam!